Effect of Internal Heat Transport on the Thermal Migration of Bubbles

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THERMAL migration has been suggested as a promising technique for bubble management in the gravity-free environment of outer space. Bubbles in a liquid whose surface tension varies with temperature are expected to migrate in a thermal gradient because of thermocapillary flow in the surface of the bubble. Several laboratory experiments have been performed which demonstrate this effect,[1-3] but experiments designed to investigate this phenomenon in the absence of gravity has given curious results. Bubbles in weightless liquids were observed to be immobile despite the presence of a thermal gradient.[4,5]

A variety of explanations for the observed immobility have been proposed; none were entirely satisfactory.[4,5] Some further laboratory demonstration experiments in which the effects of convection were considered have also been performed.[6] It is the purpose of this Note to show that, in addition to previously mentioned phenomena, the vapor pressure of the liquid and the composition of the gases in the bubble can have a profound effect on thermal transport in the bubble and, hence, on the temperature gradient on the surface of the bubble. In some cases the surface of the bubble cannot support a temperature gradient, while in other cases, the temperature gradient may be greater than that of the host liquid. Since the predicted velocity of the bubble is directly proportional to the temperature gradient on its surface,[1] situations in which this gradient is reduced by heat transport in the bubble will lead to bubble immobility.

Examples of the behavior expected for water, a silicone oil (Dow Corning 702), two organic, plastic crystal compounds (carbon tetrabromide and succinonitrile), and two metals (aluminum and magnesium) will be given. These materials were chosen as representative of the materials that have been used previously for these experiments and of materials that might be of interest for future low-gravity experimentation. The plastic crystal compounds have been successfully used for experiments that model metallic solidification[7]; one low and one high vapor pressure material were chosen. Likewise, the two metals are typical of low and high vapor pressure metals.

The temperature gradient in a spherical inclusion embedded in a host liquid will depend on the thermal conductivity of the inclusion K' and the thermal conductivity of the host K. If convection can be ignored, the thermal gradient in the inclusion V' will be given by

$$V'/V = 3K/(2K+K') \tag{1}$$

where V is the linear thermal gradient in the host.[8] Thus, if the conductivity of the embedded sphere is much less than that of host, $K' \ll K$, the thermal gradient in the sphere will be greater than that in the host, $V'/V \simeq 1.5$. Conversely, if $K' \gg K$, then the thermal gradient in the inclusion can approach zero. For the case of a small gas bubble in a liquid, several sources of thermal transport can be envisaged.

Gaseous conduction at pressures near atmospheric will result in a thermal conductivity of less than 5×10^{-2} W/mK for most gases,[9] (hydrogen and helium will be roughly three times higher). In general, this conductivity is less than the thermal conductivity of the liquid K_l, even for poor conductors such as DC702 and organic compounds. Values of K_l for the six materials are shown in Table 1. This difference between the conductivity of the gas and the liquid will result in a situation in which the surface of the bubble experiences a temperature gradient equal to or greater than that of the host liquid, i.e., $K' < K$; therefore, $V'/V \geq 1$. Thermal migration could occur.

However, for the case of a bubble which contains only vapor of the host liquid at its equilibrium vapor pressure, an additional transport mechanism can operate. Evaporation from the hot side of the bubble and condensation on the cold side can provide a potent source of heat transport and may result in a far greater apparent conductivity of the bubble. An effective thermal conductivity for this process, K_e, can be approximated as

$$K_e = (Ql)/(A\Delta T) \tag{2}$$

where l is the length, A the area, ΔT the temperature difference, and Q the amount of heat transported per second.

Q can be estimated as:

$$Q = J_n A \Delta H_v \tag{3}$$

where ΔH_v is the heat of vaporization and J_n is the net molecular flux. The evaporative flux J is given in Ref. 10 as

$$J = (P_e - P)/(2\pi MRT)^{1/2} \tag{4}$$

where P_e is the equilibrium vapor pressure of the surface at temperature T, and P is the equivalent condensation pressure. For an estimate of the net flux, we let ΔP equal the difference in equilibrium vapor pressure between the hot and cold sides of the bubble, i.e.,

$$P_e - P = \Delta P = \frac{dP_e}{dT}\frac{dT}{dx}l \tag{5}$$

and we substitute the Clausius-Clapeyron relation for dP_e/dT. Rearranging gives

$$K_e = P_e l(\Delta H_v)^2/RT^2(2\pi MRT)^{1/2} \tag{6}$$

Values of K_e calculated using this relationship for the various materials are shown in Table 2. The temperature, vapor pressure, heat of vaporization, and molecular weight used in the calculation are shown in Table 1. The temperatures were chosen to be either room temperature or slightly above the melting point of the material. The length l was assumed to be 2×10^{-3} m.

Inspection of the calculated values shows that the estimated effective thermal conductivity can vary from negligible values of about 10^{-5} W/mK in DC702 and aluminum to very large values of the order of several hundred W/mK for magnesium, carbon tetrabromide, and water. Bubbles in these high vapor materials can act as thermal short circuits. The large disparity in thermal conductivity of the liquid and the bubble causes the surface of the bubble to become nearly isothermal. Values of V'/V calculated using Eq. (1) are also listed in Table 2 to illustrate this.

The calculated values of K_e and V'/V were arrived at by way of fairly crude assumptions, and they are not intended to be highly accurate. Nevertheless, it is thought that the underlying physical concept is valid, and that in high vapor pressure materials, small bubbles that contain only the equilibrium vapor pressure of the host material will act as thermal short circuits. They are not expected to migrate in a thermal gradient because the surface of the bubble will be essentially isothermal. In the low-gravity experiments, carbon

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Table 1 Material properties

	H_2O		DC702		CBr_4		$C_4H_4N_2$		Al		Mg	
T, K	300		300		375		335		1000		1000	
M, kg/mol	1.8×10^{-2}		~0.2		0.33		8.0×10^{-2}		2.7×10^{-2}		2.4×10^{-2}	
P, N/m^2	3.5×10^3	(13)[a]	8.3×10^{-5}	(14)	6.1×10^3	(15)	20	(16)	5.4×10^{-6}	(17)	1.6×10^3	(17)
ΔH_v, J/mol	4.4×10^4	(13)	9.1×10^4	(14)	5.1×10^4	[b]	7.0×10^4	[b]	3.3×10^5	(17)	1.5×10^5	(17)
K_l, W/mK	0.55		0.13		0.2		0.22		96		81	

[a] Numbers in parentheses indicate references from which the values were obtained. [b] Derived from vapor pressure vs temperature data.

Table 2 Calculated values

	H_2O	DC702	CBr_4	$C_4H_4N_2$	Al	Mg
K_e, W/mK	1.1×10^3	3.3×10^{-5}	3.4×10^2	5.6	3.8×10^{-6}	2.4×10^2
$V'/V, K_e$	1.5×10^{-3}	1.5	1.8×10^{-3}	0.11	1.5	0.61
K_d, W/mK	1.0	2.8×10^{-5}	0.71	1.1	6.4×10^{-7}	0.32
$V'/V, K_d$	0.8	1.5	0.4	0.43	1.5	1.5

tetrabromide was used as the sample material. Table 1 shows that this was a poor choice because of its high vapor pressure. These calculations indicate that if the bubbles in the low-gravity experiment contained only CBr_4 vapor, they should not have migrated. This is consistent with the experimental observations.[4,5]

A technique for reducing the thermal conductivity of a bubble in high vapor pressure materials can be borrowed from the technology of variable conductance heat pipes. A small amount of inert gas can introduce a diffusion barrier for the vapor species and significantly reduce the thermal conductivity.[11] In order to calculate the effective conductivity of the bubble with an inert gas present, we assume J to be diffusion controlled, i.e.,

$$J_d = -D\frac{dc}{dx} \tag{7}$$

There are few data available for interdiffusion in these systems, so by comparison with other systems we assume that $D \simeq 10^{-5}$ m^2/s at 1 atm and varies inversely with P_t, the total pressure of gas in the bubble,[12] i.e., $D = 1/P_t$ where P_t is in N/m^2. Letting

$$\frac{dc}{dx} = d\frac{(n/V)}{dx} = \frac{1}{RT}\frac{dp}{dT}\frac{dT}{dx} \tag{8}$$

substituting in Eqs. (7), (3), and (2), and using the Clausius-Clapeyron equation yields

$$K_d = (P_e/P_t)(\Delta H_v)^2/R^2T^3 \tag{9}$$

Values of K_d, the effective thermal conductivity due to evaporation and condensation when some inert gas is present in the bubble, are shown in Table 2. For these calculations P_t was assumed to be the equilibrium vapor pressure plus 13.3 N/m^2 (0.1 Torr). Comparison between K_d and K_e shows that the presence of an inert gas has significantly reduced the thermal conductivity of the bubble. This will restore V'/V to values much closer to 1, as can be seen in Table 2 where values of V'/V calculated with Eq. (1) using K_l and K_d are also shown. Once again, the calculated values are not intended to be accurate, but are thought to be useful guides in analyzing the problem. They lead to the conclusion that a small amount of inert gas in a bubble can introduce a significant barrier to heat transport via an evaporation-condensation mechanism. For materials with a high vapor pressure, this might be a viable method of restoring bubble mobility in thermal gradients in low gravity.

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